Predicate Calculus for Boolean Valued Functions (13)

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二値関数における述語論理（13）

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要　旨

二値関数と集合の分割に関する述語論理について成り立つ定理を提案し、その定理の厳格な証明を行った。ここでの述語論理とは「すべての～について」や「ある～について」に関する一階述語論理を指す。本研究の目的の一つは、二値関数に集合の分割の考え方を導入することで、コンピュータの内部で行われる論理動作を、数学的にモデル化することである。デジタルの世界での論理動作を、数学的にモデル化している。

キーワード

二値関数　　述語論理　　集合の分割

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1 Abstract

In the following articles:[1], [2], [3], and [5], we have defined Boolean valued function with respect to partitions. And some of their algebraic properties are proved. We have also introduced and examined the infimum and supremum of Boolean valued functions and their properties. In this paper, some elementary Predicate calculus formulae for Boolean valued function are proved.

2 Introduction

In this paper I have proved some elementary Predicate calculus formulae for Boolean valued functions with respect to partitions. Such a theory is an analogy of usual Predicate logic.

A Boolean valued function is a function of the type $f : X \to B$, where X is a non empty set and where B is a Boolean domain. A Boolean domain B is a two element set, that is, $B = \{0, 1\}$, whose elements are interpreted as logical values, for example, $0 = false$ and $1 = true$.

The correctness of the theorems in this paper was checked with Mizar[14] proof checker of computer.

Let Y be a non empty set. The functor PARTITIONS(Y) was defined in article:[1] by:

(Def. 1) For every set x holds $x \in$ PARTITIONS(Y) iff x is a partition of Y.

The functor BVF(Y) was defined in article:[2] by :

(Def. 2) BVF(Y) $= Boolean^Y$.

Let us consider Y, let a be an element of BVF(Y), and let x be an element of Y. The functor Pj(a, x) yields an element of *Boolean* and was defined in article:[2] by:

(Def. 3) Pj$(a, x) = a(x)$.

Let us consider Y and let a, b be elements of BVF(Y). The predicate $a \Subset b$ was defined in article:[2] by:

(Def. 4) For every element x of Y such that $\mathrm{Pj}(a,x) = true$ holds $\mathrm{Pj}(b,x) = true$.

Let us consider Y and let a be an element of $\mathrm{BVF}(Y)$. The functor $INF(a)$ yields an element of $\mathrm{BVF}(Y)$ and was defined in article:[2] as follows:

(Def. 5) $INF(a) = true(Y)$, if for every element x of Y holds $\mathrm{Pj}(a,x) = true$, otherwise $false(Y)$.

The functor $SUP(a)$ yielding an element of $\mathrm{BVF}(Y)$ was defined in article:[2] by:

(Def. 6) $SUP(a) = false(Y)$, if for every element x of Y holds $\mathrm{Pj}(a,x) = false$, otherwise $true(Y)$.

Let us consider Y, let P_1 be a partition of Y, and let G be a subset of $\mathrm{PARTITIONS}(Y)$. The functor $CompF(P_1, G)$ yielding a partition of Y was defined in article:[3] by:

(Def. 7) $CompF(P_1, G) = \bigwedge G \setminus \{P_1\}$.

Let us consider Y, let a be an element of $\mathrm{BVF}(Y)$, let G be a subset of $\mathrm{PARTITIONS}(Y)$, and let P_1 be a partition of Y. The functor $\forall_{a,P_1} G$ yielding an element of $\mathrm{BVF}(Y)$ was defined in article:[3] by:

(Def. 8) $\forall_{a,P_1} G = INF(a, CompF(P_1, G))$.

Let us consider Y, let a be an element of $\mathrm{BVF}(Y)$, let G be a subset of $\mathrm{PARTITIONS}(Y)$, and let P_1 be a partition of Y. The functor $\exists_{a,P_1} G$ yielding an element of $\mathrm{BVF}(Y)$ was defined in article:[3] as follows:

(Def. 9) $\exists_{a,P_1} G = SUP(a, CompF(P_1, G))$.

3 Predicate Calculus for BVF(Y)

The terminology and notation used in this paper have been introduced in the following articles:[1], [2], [3], [5], [7], [10],and [11].

In this paper Y denotes a non empty set.

The following propositions are true:

1. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{a \Rightarrow (b \vee c), P_1} G$.

2. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{a \Rightarrow (b \vee \neg c), P_1} G$.

3. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{b \Rightarrow (c \vee a), P_1} G$.

4. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{b \Rightarrow (c \vee \neg a), P_1} G$.

5. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{(a \Rightarrow b) \wedge (b \Rightarrow (c \vee a)), P_1} G$.

6. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{(a \Rightarrow (b \vee \neg c)) \wedge (b \Rightarrow c), P_1} G$.

7. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{(a \Rightarrow (b \vee c)) \wedge (b \Rightarrow (c \vee a)), P_1} G$.

8. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \ \Subset \ \exists_{(a \Rightarrow (b \vee \neg c)) \wedge (b \Rightarrow (c \vee a)), P_1} G$.

9. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \ \Subset \ \exists_{(a \Rightarrow (b \vee \neg c)) \wedge (b \Rightarrow (c \vee \neg a)), P_1} G$.

10. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{a \wedge (a \Rightarrow b) \wedge (b \Rightarrow c), P_1} G \ \Subset \ \exists_{c, P_1} G$.

11. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \vee b) \wedge (a \Rightarrow c) \wedge (b \Rightarrow c), P_1} G \ \Subset \ \exists_{\neg a \Rightarrow (b \vee c), P_1} G$.

12. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b), P_1} G \ \Subset \ \exists_{(c \Rightarrow a) \Rightarrow (c \Rightarrow b), P_1} G$.

13. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Leftrightarrow b), P_1} G \ \Subset \ \exists_{(a \Leftrightarrow c) \Leftrightarrow (b \Leftrightarrow c), P_1} G$.

14. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Leftrightarrow b), P_1} G \ \Subset \ \exists_{(a \Rightarrow c) \Leftrightarrow (b \Rightarrow c), P_1} G$.

15. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$,

and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Leftrightarrow b),P_1} G \Subset \exists_{(c \Rightarrow a) \Leftrightarrow (c \Rightarrow b),P_1} G$.

16. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Leftrightarrow b),P_1} G \Subset \exists_{(a \wedge c) \Leftrightarrow (b \wedge c),P_1} G$.

17. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Leftrightarrow b),P_1} G \Subset \exists_{(a \vee c) \Leftrightarrow (b \vee c),P_1} G$.

18. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{a,P_1} G \Subset \exists_{(a \Leftrightarrow b) \Leftrightarrow (b \Leftrightarrow a) \Leftrightarrow a,P_1} G$.

19. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{a,P_1} G \Subset \exists_{(a \Rightarrow b) \Leftrightarrow b,P_1} G$.

20. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{a,P_1} G \Subset \exists_{(b \Rightarrow a) \Leftrightarrow a,P_1} G$.

21. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{a,P_1} G \Subset \exists_{(a \wedge b) \Leftrightarrow (b \wedge a) \Leftrightarrow a,P_1} G$.

22. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{\neg a,P_1} G \Subset \exists_{(a \Rightarrow b) \Leftrightarrow \neg a,P_1} G$.

23. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{\neg a, P_1} G \Subset \exists_{(b \Rightarrow a) \Leftrightarrow \neg b, P_1} G$.

24. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{a, P_1} G \Subset \exists_{(a \vee b) \Leftrightarrow (b \vee a) \Leftrightarrow a, P_1} G$.

25. Let a, b, c be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \vee b) \wedge (b \Rightarrow c), P_1} G \Subset \exists_{a \vee c, P_1} G$.

26. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{a \wedge (a \Rightarrow b), P_1} G \Subset \exists_{b, P_1} G$.

27. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge \neg b, P_1} G \Subset \exists_{\neg a, P_1} G$.

28. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \vee b) \wedge \neg a, P_1} G \Subset \exists_{b, P_1} G$.

29. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (\neg a \Rightarrow b), P_1} G \Subset \exists_{b, P_1} G$.

30. Let a, b be elements of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y),

and P_1 be a partition of Y. Suppose G is a coordinate and $P_1 \in G$, then $\forall_{(a \Rightarrow b) \wedge (a \Rightarrow \neg b), P_1} G \ \Subset \ \exists_{\neg a, P_1} G$.

4 Conclusion

In this paper, some elementary Predicate calculus formulae for Boolean valued function with respect to partitions are proved. The correctness of the proof was checked by Mizar[14] proof checker by using computer.

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[14] The Mizar system consists of a language for writing strictly formalized mathematical definitions and proofs, a computer program which is able to check proofs written in this language, and a library of definitions and proved theorems which can be referred to and used in new articles.

http://mizar.uwb.edu.pl