Propositional Calculus for Boolean Valued Functions (11)

KOBAYASHI Shunichi

二値関数における命題論理（11）

小林　俊一

要　　旨

二値関数と集合の分割に関する命題論理について成り立つ論理包含に関する定理を提案し、その定理の厳格な証明を行った。これらの定理は、従来の命題論理を、数学的に新しい考え方に基づいて厳格な形でモデル化して、それを定式化したものである。定理の正しさに関しては、数学証明検証システムを用いて、厳格な検証を行っている。なお、本研究の工学的な応用としては、論理回路への応用などが考えられる。

キーワード

二値関数　　命題論理　　集合の分割

目　　次

1 abstract

In the following articles:[1], [2], [4], and [5], we have defined Boolean valued function with respect to partitions. And some of their algebraic properties are proved. We have also introduced and examined the infimum and supremum of Boolean valued functions and their properties. In this paper, some elementary propositional calculus formulae for Boolean valued function are proved.

2 Introduction

In this paper I have proved some elementary propositional calculus formulae for Boolean valued functions with respect to partitions. Such a theory is an analogy of usual propositional logic.

A Boolean valued function is a function of the type $f : X \to B$, where X is a non empty set and where B is a Boolean domain. A Boolean domain B is a two element set, that is, $B = \{0, 1\}$, whose elements are interpreted as logical values, for example, $0 = false$ and $1 = true$.

The correctness of the theorems in this paper was checked with Mizar[14] proof checker of computer.

3 Propositional Calculus for $BVF(Y)$

The terminology and notation used in this paper have been introduced in the following articles:[2], [5], [7], [10],and [11].

In this paper Y denotes a non empty set and a, b, c denote elements of $BVF(Y)$. The following propositions are true:

1. $(a\ 'nand'\ b) \Rightarrow (a\ 'nor'\ b) = a \Leftrightarrow b.$

2. $(a \otimes b) \Rightarrow (a \wedge b) = a \Leftrightarrow b.$

3. $(a \otimes b) \Rightarrow (a \vee b) = true(Y).$

4. $(a \otimes b) \Rightarrow (a \Leftrightarrow b) = a \Leftrightarrow b.$

5. $(a \otimes b) \Rightarrow (a \Rightarrow b) = \neg a \vee b$.

6. $(a \otimes b) \Rightarrow (a\ 'nand'\ b) = true(Y)$.

7. $(a \otimes b) \Rightarrow (a\ 'nor'\ b) = a \Leftrightarrow b$.

8. $(a \Leftrightarrow b) \Rightarrow (a \wedge b) = a \vee b$.

9. $(a \Leftrightarrow b) \Rightarrow (a \vee b) = a \vee b$.

10. $(a \Leftrightarrow b) \Rightarrow (a \otimes b) = a \otimes b$.

11. $(a \Leftrightarrow b) \Rightarrow (a\ 'nand'\ b) = \neg a \vee \neg b$.

12. $(a \Leftrightarrow b) \Rightarrow (a\ 'nor'\ b) = \neg a \vee \neg b$.

13. $(a\ 'nor'\ b) \Rightarrow (a \wedge b) = (a \vee b) \vee (a \wedge b)$.

14. $(a\ 'nor'\ b) \Rightarrow (a \vee b) = a \vee b$.

15. $(a\ 'nor'\ b) \Rightarrow (a \otimes b) = a \vee b$.

16. $(a\ 'nor'\ b) \Rightarrow (a \Leftrightarrow b) = true(Y)$.

17. $(a\ 'nor'\ b) \Rightarrow (a \Rightarrow b) = true(Y)$.

18. $(a\ 'nor'\ b) \Rightarrow (a\ 'nand'\ b) = true(Y)$.

19. $(a \wedge b) \Rightarrow (\neg a \wedge b) = \neg a \vee \neg b$.

20. $(a \wedge b) \Rightarrow (\neg a \otimes b) = true(Y)$.

21. $(a \wedge b) \Rightarrow (\neg a \Leftrightarrow b) = \neg a \vee \neg b$.

22. $(a \wedge b) \Rightarrow (\neg a \Rightarrow b) = true(Y)$.

23. $(a \wedge b) \Rightarrow (\neg a\ 'nand'\ b) = true(Y)$.

24. $(a \wedge b) \Rightarrow (\neg a\ 'nor'\ b) = \neg a \vee \neg b$.

25. $(a \Rightarrow b) \Rightarrow (\neg a \wedge b) = a \otimes b$.

26. $(a \Rightarrow b) \Rightarrow (\neg a \otimes b) = a \vee \neg b$.

27. $(a \Rightarrow b) \Rightarrow (\neg a \Leftrightarrow b) = a \otimes b$.

28. $(a \Rightarrow b) \Rightarrow (\neg a \Rightarrow b) = a \vee b$.

29. $(a \Rightarrow b) \Rightarrow (\neg a\ 'nand'\ b) = (a \wedge \neg b) \vee (a \vee \neg b)$.

30. $(a \Rightarrow b) \Rightarrow (\neg a\ 'nor'\ b) = a \wedge \neg b$.

31. $(a \vee b) \Rightarrow (\neg a \wedge b) = \neg a$.

32. $(a \vee b) \Rightarrow (\neg a \otimes b) = a \Leftrightarrow b$.

33. $(a \vee b) \Rightarrow (\neg a \Leftrightarrow b) = \neg a \vee \neg b$.

34. $(a \vee b) \Rightarrow (\neg a \Rightarrow b) = true(Y)$.

35. $(a \vee b) \Rightarrow (\neg a\ 'nand'\ b) = a \vee \neg b$.

36. $(a \vee b) \Rightarrow (\neg a\ 'nor'\ b) = \neg b$.

37. $(a \otimes b) \Rightarrow (\neg a \wedge b) = \neg a \vee b$.

38. $(a \otimes b) \Rightarrow (\neg a \otimes b) = a \Leftrightarrow b$.

39. $(a \otimes b) \Rightarrow (\neg a \Leftrightarrow b) = true(Y)$.

40. $(a \otimes b) \Rightarrow (\neg a \Rightarrow b) = true(Y)$.

41. $(a \otimes b) \Rightarrow (\neg a\ 'nand'\ b) = a \vee \neg b.$

42. $(a \otimes b) \Rightarrow (\neg a\ 'nor'\ b) = a \vee \neg b.$

43. $(a \Leftrightarrow b) \Rightarrow (\neg a \wedge b) = a \otimes b.$

44. $(a \Leftrightarrow b) \Rightarrow (\neg a \otimes b) = true(Y).$

45. $(a \Leftrightarrow b) \Rightarrow (\neg a \Leftrightarrow b) = a \otimes b.$

46. $(a \Leftrightarrow b) \Rightarrow (\neg a \Rightarrow b) = a \vee b.$

47. $(a \Leftrightarrow b) \Rightarrow (\neg a\ 'nand'\ b) = true(Y).$

48. $(a \Leftrightarrow b) \Rightarrow (\neg a\ 'nor'\ b) = a \otimes b.$

49. $(a\ 'nand'\ b) \Rightarrow (\neg a \wedge b) = b.$

50. $(a\ 'nand'\ b) \Rightarrow (\neg a \otimes b) = a \Leftrightarrow b.$

51. $(a\ 'nand'\ b) \Rightarrow (\neg a \Leftrightarrow b) = a \vee b.$

52. $(a\ 'nand'\ b) \Rightarrow (\neg a \Rightarrow b) = (a \wedge b) \vee (a \vee b).$

53. $(a\ 'nand'\ b) \Rightarrow (\neg a\ 'nand'\ b) = a \vee \neg b.$

54. $(a\ 'nand'\ b) \Rightarrow (\neg a\ 'nor'\ b) = a.$

55. $(a\ 'nor'\ b) \Rightarrow (\neg a \wedge b) = a \vee b.$

56. $(a\ 'nor'\ b) \Rightarrow (\neg a \otimes b) = true(Y).$

57. $(a\ 'nor'\ b) \Rightarrow (\neg a \Leftrightarrow b) = a \vee b.$

4 Conclusion

In this paper, some elementary propositional calculus formulae for Boolean valued function with respect to partitions are proved. The correctness of the proof was checked by Mizar[14] proof checker by using computer.

References

[1] Shunichi Kobayashi, Kui Jia. A Theory of Partitions. Part I, Formalized Mathematics 7(2), pages 243-247, 1998.

`http://mizar.org/fm/1998-7/pdf7-2/partit1.pdf`

[2] Shunichi Kobayashi, Kui Jia. A Theory of Boolean Valued Functions and Partitions, Formalized Mathematics 7(2), pages 249-254, 1998.

`http://mizar.org/fm/1998-7/pdf7-2/bvfunc_1.pdf`

[3] Shunichi Kobayashi, Yatsuka Nakamura. A Theory of Boolean Valued Functions and Quantifiers with Respect to Partitions, Formalized Mathematics 7(2), pages 307-312, 1998.

`http://mizar.org/fm/1998-7/pdf7-2/bvfunc_2.pdf`

[4] Shunichi Kobayashi. On the Calculus of Binary Arithmetics, Formalized Mathematics 11(4), pages 417-419, 2003.

`http://mizar.org/fm/2003-11/pdf11-4/binari_5.pdf`

[5] Shunichi Kobayashi. Propositional Calculus for Boolean Valued Functions. Part VIII, Formalized Mathematics 13(1), pages 55-58, 2005.

`http://mizar.org/fm/2005-13/pdf13-1/bvfunc26.pdf`

[6] Czeslaw Bylinski. Functions and Their Basic Properties, Formalized Mathematics 1(1), pages 55-65, 1990.

`http://mizar.org/fm/1990-1/pdf1-1/funct_1.pdf`

[7] Andrzej Trybulec. Function Domains and Fraenkel Operator, Formalized Mathematics 1(3), pages 495-500, 1990.

`http://mizar.org/fm/1990-1/pdf1-3/fraenkel.pdf`

[8] Andrzej Trybulec. Tarski Grothendieck Set Theory, Formalized Mathematics 1(1), pages 9-11, 1990.

`http://mizar.org/fm/1990-1/pdf1-1/tarski.pdf`

[9] Zinaida Trybulec. Properties of Subsets, Formalized Mathematics 1(1), pages 67-71, 1990.

`http://mizar.org/fm/1990-1/pdf1-1/subset_1.pdf`

[10] Edmund Woronowicz. Interpretation and Satisfiability in the First Order Logic, Formalized Mathematics 1(4), pages 739-743, 1990.

`http://mizar.org/fm/1990-1/pdf1-4/valuat_1.pdf`

[11] Edmund Woronowicz. Many–Argument Relations, Formalized Mathematics 1(4), pages 733-737, 1990.

`http://mizar.org/fm/1990-1/pdf1-4/margrel1.pdf`

[12] Edmund Woronowicz. Relations and Their Basic Properties, Formalized Mathematics 1(1), pages 73-83, 1990.

`http://mizar.org/fm/1990-1/pdf1-1/relat_1.pdf`

[13] Grzegorz Bancerek, Andrzej Trybulec. Miscellaneous Facts about Functions, Formalized Mathematics 5(4), pages 485-492, 1996.

`http://mizar.org/fm/1996-5/pdf5-4/funct_7.pdf`

[14] The Mizar system consists of a language for writing strictly formalized mathematical definitions and proofs, a computer program which is able to check proofs written in this language, and a library of definitions and proved theorems which can be referred to and used in new articles.

`http://mizar.org`