# Propositional Calculus for Boolean Valued Functions (9)

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## 1 abstract

In the following articles:[ 1 ], [ 2 ], [ 4 ], and [ 5 ], we have defined Boolean valued function with respect to partitions. And some of their algebraic properties are proved. We have also introduced and examined the infimum and supremum of Boolean valued functions and their properties. In this paper, some elementary propositional calculus formulae for Boolean valued function are proved.


## 2 Introduction

In this paper I have proved some elementary propositional calculus formulae for Boolean valued functions with respect to partitions. Such a theory is an analogy of usual propositional logic.

A Boolean valued function is a function of the type $f : X \to B$, where $X$ is a non empty set and where $B$ is a Boolean domain. A Boolean domain $B$ is a two element set, that is, $B = \{0, 1\}$, whose elements are interpreted as logical values, for example, $0 = false$ and $1 = true$.

The correctness of the theorems in this paper was checked with Mizar[14] proof checker of computer.

## 3 Propositional Calculus for $BVF(Y)$

The terminology and notation used in this paper have been introduced in the following articles:[ 2 ], [ 5 ], [ 7 ], [10], and [11].

In this paper $Y$ denotes a non empty set and $a$, $b$, $c$ denote elements of $BVF(Y)$. The following propositions are true:

1. $a \wedge (a \wedge b) = a \wedge b$.
2. $a \wedge (a \vee b) = a \vee (a \wedge b)$.
3. $\neg a \wedge (a \wedge b) = false(Y)$.
4. $\neg a \wedge (a \vee b) = \neg a \wedge b$.
5. $\neg a \vee (a \wedge b) = \neg a \vee b$.
6. $\neg a \vee (a \vee b) = true(Y)$.
7. $\neg a \vee (a \otimes b) = \neg a \vee \neg b$.
8. $\neg a \vee (a\ 'nor'\ b) = \neg a$.

9. $\neg a \wedge \neg(a \wedge b) = \neg a$.

10. $\neg a \otimes (a \wedge b) = \neg a \vee b$.

11. $\neg a \otimes (a \vee b) = a \vee \neg b$.

12. $a \wedge (a \otimes b) = a \wedge \neg b$.

13. $\neg a \wedge (a \Leftrightarrow b) = \neg a \wedge \neg b$.

14. $\neg a \otimes (a \otimes b) = \neg b$.

15. $\neg a \otimes (a\ '\mathit{nand}'\ b) = a \wedge \neg b$.

16. $\neg a \otimes (a\ '\mathit{nor}'\ b) = \neg a \wedge b$.

17. $a \vee (a \vee b) = a \vee b$.

18. $\neg a \vee \neg(a \wedge b) = \neg a \vee \neg b$.

19. $\neg a \vee \neg(a \vee b) = \neg a$.

20. $\neg a \Leftrightarrow (a \wedge b) = a \wedge \neg b$.

21. $\neg a \Leftrightarrow (a \vee b) = \neg a \wedge b$.

22. $\neg a \vee (a \Leftrightarrow b) = \neg a \vee b$.

23. $a \vee (a \otimes b) = a \vee b$.

24. $\neg a \vee \neg(a \otimes b) = \neg a \vee b$.

25. $\neg a \Leftrightarrow (a \otimes b) = b$.

26. $a \vee (b \Leftrightarrow c) = (a \vee \neg b \vee c) \wedge (a \vee \neg c \vee b)$.

27. $a \vee (a \Leftrightarrow b) = a \vee \neg b$.

28. $\neg(a \Leftrightarrow b) = (a \wedge \neg b) \vee (b \wedge \neg a)$.

29. $\neg a \vee \neg(a \Leftrightarrow b) = \neg a \vee \neg b$.

30. $\neg a \Leftrightarrow (a \Leftrightarrow b) = \neg b$.

31. $\neg a \vee \neg(a \Rightarrow b) = \neg a \vee \neg b$.

32. $\neg a \Leftrightarrow (a \Rightarrow b) = \neg a \vee \neg b$.

33. $\neg a \vee \neg(a\ '\mathit{nand}'\ b) = \neg a \vee b$.

34. $\neg a \Leftrightarrow (a\ '\mathit{nand}'\ b) = \neg a \vee b$.

35. $\neg a \vee \neg(a\ '\mathit{nor}'\ b) = \mathit{true}(Y)$.

36. $\neg a \Leftrightarrow (a\ '\mathit{nor}'\ b) = a \vee \neg b$.

37. $\neg a \Rightarrow (a \wedge b) = a$.

38. $\neg a \Rightarrow (a \vee b) = a \vee b$.

39. $\neg a \Rightarrow (a \otimes b) = a \vee b.$

40. $\neg a \Rightarrow (a \Leftrightarrow b) = \neg b \vee a.$

41. $\neg a \Rightarrow (a \text{ 'nand' } b) = true(Y).$

42. $\neg a \Rightarrow (a \text{ 'nor' } b) = a \vee \neg b.$

43. $\neg a \text{ 'nand' } (a \wedge b) = true(Y).$

44. $\neg a \text{ 'nand' } (a \vee b) = a \vee \neg b.$

45. $\neg a \text{ 'nand' } (a \otimes b) = a \vee \neg b.$

46. $\neg a \text{ 'nand' } (a \Leftrightarrow b) = a \vee b.$

47. $\neg a \wedge (a \Rightarrow b) = \neg a.$

48. $\neg a \text{ 'nand' } (a \Rightarrow b) = a.$

49. $\neg a \wedge (a \text{ 'nand' } b) = \neg a.$

50. $\neg a \text{ 'nand' } (a \text{ 'nand' } b) = a.$

51. $\neg a \wedge (a \text{ 'nor' } b) = \neg a \wedge \neg b.$

52. $\neg a \text{ 'nand' } (a \text{ 'nor' } b) = a \vee b.$

53. $false(Y) \Leftrightarrow a = \neg a.$

54. $a \wedge (b \Leftrightarrow c) = a \wedge (\neg b \vee c) \wedge (\neg c \vee b).$

55. $a \wedge (a \Leftrightarrow b) = a \wedge b.$

56. $a \vee (b \otimes c) = a \vee (\neg b \wedge c) \vee (b \wedge \neg c).$

57. $a \vee (b \text{ 'nor' } c) = (a \vee \neg b) \wedge (a \vee \neg c).$

58. $a \vee (b \text{ 'nor' } c) = (b \Rightarrow a) \wedge (c \Rightarrow a).$

59. $a \Rightarrow (b \otimes c) = \neg a \vee (\neg b \wedge c) \vee (b \wedge \neg c).$

60. $a \Rightarrow (b \Leftrightarrow c) = (\neg a \vee \neg b \vee c) \wedge (\neg a \vee b \vee \neg c).$

61. $a \Rightarrow (b \text{ 'nand' } c) = \neg a \vee \neg b \vee \neg c.$

62. $a \Rightarrow (b \text{ 'nor' } c) = (\neg a \vee \neg b) \wedge (\neg a \vee \neg c).$

63. $a \Rightarrow (b \text{ 'nor' } c) = (a \Rightarrow \neg b) \wedge (a \Rightarrow \neg c).$

64. $a \Rightarrow (a \otimes b) = \neg a \vee \neg b.$

65. $a \Rightarrow (a \Rightarrow b) = \neg a \vee b.$

66. $a \Rightarrow (a \Rightarrow b) = a \Rightarrow b.$

67. $a \Rightarrow (a \Leftrightarrow b) = a \Rightarrow b.$

68. $a \Rightarrow (a \Leftrightarrow b) = a \Rightarrow (a \Rightarrow b).$

69. $a \Rightarrow (a \text{ 'nor' } b) = \neg a$.

70. $a \text{ 'nand' } (b \Leftrightarrow c) = \neg((a \wedge (\neg b \vee c)) \wedge (\neg c \vee b))$.

71. $a \text{ 'nand' } (b \text{ 'nand' } c) = (\neg a \vee b) \wedge (\neg a \vee c)$.

72. $a \text{ 'nand' } (b \text{ 'nand' } c) = (a \Rightarrow b) \wedge (a \Rightarrow c)$.

73. $a \text{ 'nand' } (a \otimes b) = \neg a \vee b$.

74. $a \text{ 'nand' } (a \otimes b) = a \Rightarrow b$.

75. $a \text{ 'nor' } (b \otimes c) = \neg(a \vee (\neg b \wedge c) \vee (b \wedge \neg c))$.

76. $a \text{ 'nor' } (b \Leftrightarrow c) = \neg((a \vee \neg b \vee c) \wedge (a \vee \neg c \vee b))$.

77. $a \text{ 'nor' } (a \wedge b) = \neg a$.

78. $a \text{ 'nor' } (a \otimes b) = \neg a \wedge \neg b$.

79. $a \otimes (b \wedge c) = (a \vee (b \wedge c)) \wedge (\neg a \vee \neg(b \wedge c))$.

80. $a \otimes (a \wedge b) = a \wedge \neg b$.

81. $a \otimes (a \vee b) = \neg a \wedge b$.

82. $a \wedge \neg(a \otimes b) = a \wedge b$.

83. $a \otimes (a \otimes b) = b$.

84. $a \wedge \neg(a \Rightarrow b) = a \wedge \neg b$.

85. $a \otimes (a \Rightarrow b) = \neg a \vee \neg b$.

86. $a \wedge \neg(a \Leftrightarrow b) = a \wedge \neg b$.

87. $a \otimes (a \Leftrightarrow b) = \neg b$.

88. $\neg a \wedge (b \Leftrightarrow c) = \neg a \wedge (\neg b \vee c) \wedge (\neg c \vee b)$.

89. $\neg a \vee (a \Rightarrow b) = \neg a \vee b$.

90. $\neg a \vee (a \text{ 'nand' } b) = \neg a \vee \neg b$.

91. $\neg a \otimes (a \Rightarrow b) = a \wedge b$.

92. $\neg a \otimes (b \Rightarrow a) = a \vee b$.

93. $\neg a \otimes (a \Leftrightarrow b) = b$.

## 4 Conclusion

In this paper, some elementary propositional calculus formulae for Boolean valued function with respect to partitions are proved. The correctness of the proofs was checked by Mizar[14] proof checker by using computer.

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[14] The Mizar system consists of a language for writing strictly formalized mathematical definitions and proofs, a computer program which is able to check proofs written in this language, and a library of definitions and proved theorems which can be referred to and used in new articles.
http://mizar.org